# AESB 2320 Pt. 1 Exam 2016-17

1. This problem is identical to the Newtonian fluid in a tube up to the point of integrating $dv_z/dr$. Eq 2.3-14 in BSL1, [2.3-16 in BSLK, 2.3-15 in BSL2]:

$$\frac{dv_z}{dr} = -\left(\frac{\mathcal{P}_0 - \mathcal{P}_L}{2\mu L}\right) r = -\left(\frac{\mathcal{P}_0 - \mathcal{P}_L}{2L}\right)\left(A + B\frac{r}{R}\right) r = -\frac{\mathcal{P}_0 - \mathcal{P}_L}{2L}\left(Ar + B\frac{r^2}{R}\right)$$

$$v_z = -\left(\frac{\mathcal{P}_0 - \mathcal{P}_L}{2L}\right)\left[A\frac{r^2}{2} + B\frac{r^3}{3R}\right] + C_2$$

B.C.: $v_z = 0$ at $r = R \rightarrow$

$$0 = -\left(\frac{\mathcal{P}_0 - \mathcal{P}_L}{2L}\right)\left[A\frac{R^2}{2} + \frac{BR^2}{3}\right] + C_2$$

$$C_2 = \left(\frac{\mathcal{P}_0 - \mathcal{P}_L}{2L}\right)\left[A\frac{R^2}{2} + \frac{BR^2}{3}\right]$$

Plugging back into equation for $v_z$,

$$v_z = \left(\frac{\mathcal{P}_0 - \mathcal{P}_L}{2L}\right)\left[A\left(\frac{R^2 - r^2}{2}\right) + \frac{B(R^3 - r^3)}{3R}\right]$$

2. Take inlet surface 1 in the tank just before the tube entrance, surface 2 just past the end of the pipe. Eq. 7.5-11 and 7.5-12 (BSLK) are identical if one realizes $D_h = 4R_h = D$ for circular pipes. (Eq. 7.5-10 in BSL2)

$\frac{1}{2}(v_2^2 - v_1^2) = \frac{1}{2}[v^2 - 0]$ $\quad v$ = ave velocity in pipe; vel. in tank $= 0$.

$g(h_2 - h_1) = 9.8(2)$ $\quad$ outlet 2 m higher than inlet.

$$\int_{p_1}^{p_2} \frac{1}{\rho}dp = \frac{p_2 - p_1}{\rho} = \frac{10^5 - 5\cdot 10^5}{1000} = 400$$

$\hat{W}_m = 0$

3 lengths of pipe of dia. 0.003 m.

$$\sum 2v^2 \frac{L}{D} f = 2v^2 \frac{3}{0.003} f(Re) = 2000 v^2 f(Re)$$

$$Re = \frac{Dv\rho}{\mu} = \frac{0.003 v (1000)}{0.001} = 3000 v; \quad \frac{k}{D} = 0.004$$

fittings: 2 square 90° elbows $(e_v \doteq 1.6)$

1 sudden contraction $\quad e_v = 0.45(1-\beta) \simeq 0.45$

$$\sum_i \left(\frac{1}{2}v^2 e_v\right) = \frac{1}{2}v^2(1.6 + 1.6 + 0.45) = 1.825 v^2$$

Putting it all together:

$$\frac{1}{2}v^2 + 19.6 - 400 = -2000 v^2 f - 1.825 v^2$$

$$v^2\left(\frac{1}{2} + 2000 f + 1.825\right) = 400 - 19.6 = 380.4$$

$$v = \left[380.4 / (2.325 + 2000 f)\right]^{1/2}$$

Proceed by trial + error.

Guess $v = 1$ m/s. $Re = 3000$; fr. Fig 6.2-2 $f \approx 0.011$; $V = \frac{(380.4)^{1/2}}{(2.325 + 2000(0.011))^{1/2}}$

$V = 3.96$ m/s $\quad 1.19 \cdot 10^4 \quad 0.009 \quad V = 4.33$

$4.33 \quad 1.29 \cdot 10^4 \quad 0.009$

can't see any difference, done

$V \cong 4.33$ m/s

3. $K = \frac{D_p^2}{150} \frac{\varepsilon^3}{(1-\varepsilon)^2}$

coarse: $3 \cdot 10^{-10} = \frac{D_p^2}{150} \frac{(0.42)^3}{(0.58)^2} \rightarrow D_p = 4.52 \cdot 10^{-4}$ (0.45 mm)

fine: $6 \cdot 10^{-12} = \frac{D_p^2}{150} \frac{(0.42)^3}{(0.58)^2} = 6.39 \cdot 10^{-5}$ (0.064 mm)

[Note, for fixed porosity, $K \sim D_p^2$]

4. a) $Q \propto \Delta\mathcal{P}/L$ NOT

b) Turbulence makes $(\Delta\mathcal{P}/L)$ rise more than proportionately to $Q$, or $Q$ to rise less than proportionately to $(\Delta\mathcal{P}/L)$. (On homework, we showed that for highly turbulent flow in a rough pipe, $v \sim (\Delta\mathcal{P}/L)^{1/2}$. YES

c) Once minimum $(\Delta\mathcal{P}/L)$ is exceeded, $Q$ rises more than proportionately to $(\Delta\mathcal{P}/L)$. NOT

d) $Q \propto (\Delta\mathcal{P}/L)^{1/n}$, with $(1/n) > 1$. NOT

e) $Q \propto (\Delta\mathcal{P}/L)^{1/n}$ " $(1/n) < 1$. YES

f) $(\Delta\mathcal{P}/L) = \frac{\Delta p - \rho g h}{L}$. If $\Delta p$ doubles, $(\Delta p - \rho g h)$ more than doubles. $Q \sim (\Delta\mathcal{P}/L)$. NOT

g) If $\Delta p$ doubles, $(\Delta p + \rho g h)$ increases less than 2x. So $Q$ increases less than 2x. YES

h) $Q \sim B^3(\Delta\mathcal{P}/L)$. If $B$ were larger in the second test, $(\Delta\mathcal{P}/L)$ rises less than proportionately to $Q$. NOT.